

1. [16] Find the following limits if they exist, or show that they do not exist (Hint: You may use L'Hopital's rule whenever needed)

a. $\lim_{x \rightarrow +\infty} \frac{1 + 2x^3 - 3x}{x + 4 - 5x^2}$ _____/3pts

b. $\lim_{x \rightarrow 3^+} \frac{2x - 6}{\ln(x^2 - 8)}$ _____/4pts

c. $\lim_{x \rightarrow (-2)} \frac{x - 2}{x^2 + 2x}$ _____/4pts

d. $\lim_{x \rightarrow 0^+} (5x + \cos x)^{\frac{3}{x}}$ _____/5pts

2. [8] Given $f(x) = \begin{cases} \frac{x-2}{\sqrt{x+7}-3} & , \text{ if } x > 2 \\ 5x+b & , \text{ if } x = 2 \\ 3+ax & , \text{ if } 0 < x < 2 \\ 3+e^{2x} & , \text{ if } x \leq 0 \end{cases}$

a. Is $f(x)$ continuous at $x=0$? Justify.

____/3pts

b. Find the values of a and b to make $f(x)$ continuous at $x=2$.

____/5pts

3. [18] Find the derivatives of the following functions. YOU DON'T HAVE TO SIMPLIFY.

a. $f(x) = (x^2 + \sqrt{x} + 4e^2) \tan(3x)$ _____/4pts

b. $f(x) = \frac{x + e^{2x}}{4x^3 - \ln x}$ _____/4pts

c. $f(x) = \frac{\sqrt[3]{3}}{x} + 3^{x^2} + \sec(x^3)$ _____/5pts

d. $f(x) = (2 \sin x + 7)^{x-x^2}$ _____/5pts

4. [7] Given $f(x) = 1 + \sqrt[3]{6x^2 - 2x^3}$, find all the critical points then find the absolute maximum and absolute minimum on the interval $[0,3]$

5. [10] Find the derivative y' (i.e. $\frac{dy}{dx}$) in the following cases

a. $y = \sqrt{5 + \cos(1 + 4x^3)}$ _____/4pts

b. $e^{x^2} + y \sin y = 2x y^3$ _____/6pts

6. [10] Given the function $f(x) = (x^2 - 4x + 2)e^{2x}$

a. Find the critical points of $f(x)$

____/4pts

b. Find all the local maximum and minimum values of $f(x)$

____/3pts

c. Write the linear approximation of $f(x)$ at $x = 1$

____/3pts

7. [18] Evaluate the following integrals

a. $\int_1^2 2x^3 + \frac{1}{x^2} dx$ _____/4pts

b. $\int \frac{\sin(\ln x)}{2x} dx$ _____/4pts

c. $\int \frac{x + e^{-2x}}{x^2 - e^{-2x}} dx$ _____/5pts

d. $\int_0^2 5x^2 e^{8-x^3} dx$ _____/5pts

8. [5] Find $f(x)$, given that $f'(x) = 4x - \frac{5}{x} + \frac{1}{\sqrt{x}}$, and $f(1) = 7$

9. [8] Find the area between the curves $f(x) = x^2 + 5x + 2$ and $g(x) = 3x^2 - x + 2$.